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Relation

- order pairs
- cartesian product of two sets
- Relation | Binary Relation
- operations on Relation

<u>order pairs</u> - consist of two elements such that one of them is designated as first member and other as second member: (a,b)

If p is first element and q is the second member then the order pair is written as (p,q)

<u>Cartesion of two sets</u> A and B is the set of all order pairs whose first member belongs to set A and second member belongs set B and denoted by $A \times B$

$$\boxed{A \times B = \{(x,y) : x \in A \text{ and } y \in B\}}$$

Ex $A = \{a_1, a_2, \ldots a_k\}$

$B = \{b_1, b_2, \ldots b_m\}$

$A \times B = \{(a_1, b_1), (a_1, b_2), \ldots, (a_1, b_m) \ldots (a_k, b_m)\}$

Note - $n(A \times B) = |A \times B| = m \cdot n$

Ex $A = \{1, 2\}$

$B = \{3, 4, 5\}$

$A \times B = \{(1,3), (1,4), (1,5)$
$(2,3), (2,4), (3,5)\}$

$n(A) = 2$

$n(B) = 3$

$\boxed{|A \times B| = 2 \times 3 = 6}$ No. of element.

Ex $P = \{a, b, c\}$ $P \times P = ?$

$P = \{a, b, c\}$

$P \times P = \{(a,a), (a,b), (a,c),$
$(b,a), (b,b), (b,c)$
$(c,a), (c,b), (c,c)$
$\}$

$|P \times P| = |P|^2 \Rightarrow 3^2 = 9$

Relation or Binary relation -

Let A and B are two sets, A binary relation from A to B is a subset of cartesian product $A \times B$

Let $\boxed{R \subseteq A \times B}$ means R is a set of ordered pairs of the form (a,b) where $a \in A$ and $b \in B$.

- we use the notation $a R b$ to denote $(a,b) \in R$

  $a \not{R} b$ to denote $(a,b) \in R$,

  if $a R b$ we say that a is related to B by R

Ex. $A = \{a,b,c\}$ $B = \{1,2,3\}$

(i) is $R = \{(a,1), (b,2), (c,3)\}$ a relation from A to B ? Yes

(ii) $Q = \{(1,a), (2,b)\}$ is a relation from A to B ? NO

(iii) $P = \{(a,a), (b,c), (b,a)\}$ a relation from A to A ? Yes

$\because A \times B = \{(a,1), (a,2), (a,3)$
$(b,1), (b,2), (b,3)$
$(c,1), (c,2), (c,3)\}$

$n(A) = 3$
$n(B)$ 3

$\therefore$ Total order pair is 3×3
$= 9$

$A = \{a, b, c\}$

$A = \{a, b, c\}$

$A \times A = \{(a,a), (a,b), (a,c)$
$(b,a), (b,b), (b,c)$
$(c,a), (c,b), (c,c)\}$

Note -

If R is subset of set of cartesian product then

R is relation from $\boxed{A \text{ to } B \Rightarrow R \subseteq A \times B}$

or

$\boxed{R = \{(x,y) ; x \in A, y \in B \text{ and } xRy\} \subseteq A \times B}$

where xRy denotes that x is Related to y by R

$x \not R y$ " " x is not related to y

Ex Let $A = \{1, 2, 5\}$

$B = \{2, 4\}$

Find out the relation from A to B defined by "is less than"

Solⁿ $A = \{1, 2, 5\}$

$B = \{2, 4\}$

$A \times B = \{(1,2), (1,4), (2,2), (2,4), (5,2), (5,4)\}$

Lets take 1st pair

$(1,2)$ $1<2 \Rightarrow 1R2 \Rightarrow (1,2) \in R$

$(1,4)$ $1<4 \Rightarrow 1R4 \Rightarrow (1,4) \in R$

$(2,4)$ $2<4 \Rightarrow 2R4 \Rightarrow (2,4) \in R$

$(5,2)$ $5>2 \Rightarrow 5 \not R 2$

$(5,4)$ $5>4 \Rightarrow 5 \not R 4$

Therefore $\boxed{R = \{(1,2),(1,4),(2,4)\}}$

Domain and Range of Relation—

Let $R = \{(x,y) ;\ x \in A, y \in B \text{ and } xRy\}$

Domain of R = set of 1st co-ordinates of every element of R.

Range of R = " " 2nd " " " " "

(x, y)

Ex- $R = \{(1,2),(1,4),(2,4)\}$

Domain = $\{1,2\}$

Range = $\{2,4\}$

Note –

Total No. of distinct binary relations from set A to B

$= 2^{|A \times B|} = 2^{|A| \times |B|} = 2^{m.n}$ where

$|A|$ or $n(A) = m$

$|B|$ or $n(B) = n$

Relation on a set –

$R \subseteq A \times A$

or $R = \{(x, y) ; x, y \in A \text{ and } xRy\}$

Ex $A = \{1, 2, 3\}$

Find the relation on A defined by " $\leq$ "

Soln –

$A = \{1, 2, 3\}$

$A = \{1, 2, 3\}$

$A \times A = \{(1,1), (1,2), (1,3)$

$(2,1), (2,2), (2,3)$

$(3,1), (3,2), (3,3)\}$

$$\boxed{R = \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}}$$

Identity Relation — I_A on a set A is defined by

$$I_A = \{(x,y) ; x,y \in A \text{ and } x = y\}$$
$$= \{(x,x) ; x \in A\}$$

Ex $A = \{a, b, c\}$

$$I_A = \{(a,a), (b,b), (c,c)\}$$

operations on relations —

complement Relation —

consider a relation R from set A to B complement of Relation denoted by R' or $\overline{R}$ is relation from A to B such that

$$\boxed{\overline{R} \text{ or } R' = \{(a,b) ; (a,b) \notin R\}}$$

Ex Let R is relation from set X to Y

where $X = \{1, 2, 3\}$

$Y = \{8, 9\}$

and $R = \{(1,8), (2,8), (1,9), (3,9)\}$

Find complement of R ⇒ ?

Soln

$$X \times Y = \{(1,8), (1,9), (2,8), (2,9), (3,8), (3,9)\}$$

$$\overline{R} = \{(a,b) ; (a,b) \notin R\}$$

$$\overline{R} = \{(2,9), (3,8)\}$$

$\bar{\mathcal{B}}$ = Inverse Relation –

Let R be a relation from A to B, the inverse relation R denoted by R^{-1} is a relation from B to A such that

$$\boxed{R^{-1} = \{(b,a) : (a,b) \in R\}}$$

$\because R = \{a, b\}$

Ex Find R^{-1} to R on set A defined by " x+y divisible by 2" for A = {1, 2, 3}

Soln –

A = {1, 2, 3}

A = {1, 2, 3}

A x A = {(1, 1), (1, 2), (1, 3),
(2, 1), (2, 2), (2, 3)
(3, 1), (3, 2), (3, 3)}

$\because$ (1, 1)
$\Rightarrow$ 1+1=2
divisible by 2

R = {(1, 1), (1, 3), (2, 2), (3, 1), (3, 3)}

R^{-1} = {(1, 1), (3, 1), (2, 2), (1, 3), (3, 3)}

Intersection and union of Relation –

if R and S are two relations then

$$\boxed{\begin{array}{l} R \cup S = \{(x,y) ;\ xRy \text{ or } xSy\} \\ R \cap S = \{(x,y) ;\ xRy \text{ and } xSy\} \end{array}}$$

Ex

$R_1 = \{(1,1), (2,2), (3,3), (4,4), (3,4), (4,3)\}$

$R_2 = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1)\}$

$R_1 \cup R_2 = \{(1,1), (2,2), (3,3), (4,4), (3,4), (4,3), (1,2), (2,1)\}$

$R_1 \cap R_2 = \{(1,1), (2,2), (3,3), (4,4)\}$

Properties of Relation—

① Reflexive Relation

A relation on set A is reflexive if $\boxed{(a,b) \in R \ \forall\ a \in A}$

or

$\boxed{aRa \ \forall\ a \in A}$

Ex $A = \{a, b\}$

$R = \{(a,a), (a,b), (b,b)\}$

$\left.\begin{array}{l} (a,a) \in R \Rightarrow aRa \\ (b,b) \in R \Rightarrow bRb \end{array}\right\}$ then R is reflexive

Ex $A = \{a, b\}$

$R = \{(a,a), (b,a)\}$

$\left.\begin{array}{l} (a,a) \in R \Rightarrow aRa \\ (b,b) \notin R \Rightarrow b \not{R} b \end{array}\right\}$ so R is not reflexive.

② Irreflexive Relation—

any Relation R is irreflexive if $\boxed{(a,a) \notin R \ \forall\ a \in A = a \not{R} a \ \forall\ a \in A}$

Ex Let $A = \{1, 2\}$

$R = \{(1,2), (2,1)\}$

$\left.\begin{array}{l} \because (1,1) \notin R \\ (2,2) \notin R \end{array}\right\} \rightarrow$ so R is irreflexive.

③ **Non-reflexive Relation** –

A relation R on set A is non-reflexive relation if R is neither reflexive nor irreflexive.

④ **Symmetric Relation** –

if R is a relation in the set A, then R is called symmetric relation if "a is related to b then b is also related to a".

i.e. $\boxed{\begin{array}{l}(a,b) \in R \text{ Then } (b,a) \in R \\ \quad\text{or} \\ aRb \Rightarrow bRa\end{array}}$

Note - The relation R will be symmetric if $\boxed{R = R^{-1}}$

Ex-1 $A = \{2, 4, 5, 6\}$

$R_1 = \{(2,4), (4,2), (4,5), (5,4), (6,6)\}$

$(2,4) \in R \Rightarrow (4,2) \in R$

$(4,5) \in R \Rightarrow (5,4) \in R$

$(6,6) \in R \Rightarrow (6,6) \in R$

So R_1 is symmetric relation.

Ex-2 $R_2 = \{(2,4), (2,6), (6,2), (5,4), (4,5)\}$

R_2 is symmetric or not?

Sol^n - Let

$(2,4) \in R_2 \Rightarrow (4,2) \notin R_2$

So R_2 is not symmetric

Antisymmetric Relation -

A Relation R said to be antisymmetric if

$(a,b) \in R$ and $(b,a) \in R \Rightarrow a=b$

aRb and $bRa \Rightarrow a=b$

Ex In the set of Natural numbers, the relation "a divides b" is antisymmetric

Ex - if $\frac{4}{2}$ is $\frac{2}{4} \Rightarrow$ True / False $\Rightarrow$?

Note - Since a divides b and b divide a is possible only when $a=b$

i.e a/b & $b/a \Rightarrow a=b$

i.e aRb & $bRa \Rightarrow a=b$] This is condition of antisymmetric relation.

Asymmetric Relation –

A relation R on set A is asymmetric if

$$\boxed{\begin{array}{l}(a,b) \in R \text{ then } (b,a) \notin R \\ \text{i.e. } aRb \text{ then } b \not R a\end{array}}$$

→ when $\boxed{a \neq b}$

Ex $A = \{1, 2, 3\}$

$R = \{(1,2), (1,1), (2,3), (3,1)\}$

Let take

$(1,2)$ $(1,2) \in R$ then $(2,1) \notin R$

$(2,3)$ $(2,3) \in R$ " $(3,2) \notin R$

$(3,1)$ $(3,1) \in R$ " $(1,3) \notin R$

condition statisfied

Transitive Relation –

condition

$$\boxed{\begin{array}{l}\text{if } (a,b) \in R \text{ and } (b,c) \in R \\ \text{Then } (a,c) \in R\end{array}}$$

i.e. $\boxed{aRb \text{ and } bRc \Rightarrow aRc \ \forall a,b,c \in A}$

Ex-1 $A = \{1, 3, 5\}$, $R = \{(1,3), (1,5), (3,5)\}$

Let $(1,3) \in R$ and $(3,5) \in R \Rightarrow (1,5) \in R$

∴ R satisfy transitive property

Ex-2 $A = \{3, 4, 5\}$

$R = \{(3,4), (4,3), (5,4), (5,3)\}$

Check transitive property?

Soln

Let $(3,4) \in R$ and $(4,3) \in R$

so $(3,3) \notin R$

condition not satisfy

Equivalene Relation and ~~Equivalence class~~

- **Equivalence Relation** – Let A be the non-empty set and R be a relation defined on A. Then R is said to be equivalence relation if it is – fulfill
  - Reflexive
  - Symmetric
  - Transitive condition

in Reflexive –

$(a,a) \in R \ \forall a \in A$

$aRa \ \forall \ a \in A$

summetric –

$(a,b) \in R \Rightarrow (b,a) \in R \ \cancel{\forall a,b} \ \forall a,b \in A$

$aRb \Rightarrow bRa$

Transitive –

$(a,b) \in R$ and $(b,c) \in R$

Then $(a,c) \in R$

Equivalence Class –

consider an equivalence relation R on a set A. The equivalence class of an element $\boxed{a \in A}$ is the set of elements of A to which element a is related. it is denoted by $[a]$ or $\bar{a}$

Hence

$[a] = \{ b \in A : aRb \}$

Ex-1 $A = \{0, 1, 2, 3, 4\}$

$R = \{ (0,0), (0,4), (1,1), (1,3), (2,2), (3,1), (3,3), (4,0), (4,4) \}$ is an equivalence relation

Find the distinct equivalence class of R.

Soln — ① Reflexive property — for all element for set A

Let $0 \to \because (0,0) \in R$
$1 \to (1,1) \in R$
$2 \to (2,2) \in R$
$3 \to (3,3) \in R$
$4 \to (4,4) \in R$

$\to \because (a,a) \in R \ \forall a \in A$
So the Relation R fulfill the reflexive property.

② Symmetric property.

$(a,b) \in R \Rightarrow (b,a) \in R$
$(0,4) \in R \Rightarrow (4,0) \in R$
$(1,3) \in R \Rightarrow (3,1) \in R$
$(0,0) \in R \Rightarrow (0,0) \in R$
$(1,1) \in R \Rightarrow (1,1) \in R$
$(2,2) \in R \Rightarrow (2,2) \in R$
$(3,3) \in R \Rightarrow (3,3) \in R$
$(4,4) \in R \Rightarrow (4,4) \in R$

$\because (a,b) \in R \Rightarrow (b,a) \in R$
R satisfy the symmetric property

(3) <u>Transitive property</u> —

(a, b) ∈ R and (b, c) ∈ R $\Rightarrow$ then (a, c) ∈ R ∀ a, b,

~~(0, 0) ∈ R~~

(1, 3) ∈ R and (3, 1) ∈ R $\Rightarrow$ (1, 1) ∈ R ✓

(0, 4) ∈ R and (4, 0) ∈ R $\Rightarrow$ (0, 0) ∈ R ✓

so it fulfill the transitive property

Here <u>R is equivalence relation</u>.

<u>equi class</u>

$[0] = \{0, 4\}$

$[1] = \{1, 3\}$.

$[2] = \{2\}$

$[3] = \{1, 3\}$

$[4] = \{0, 4\}$

Ex-2: Let $X = \{1, 2, 3, \ldots, 7\}$

$R = \{(x, y) : (x-y)$ is divisible by $3\}$

Show that R is equivalence relation.

[AKTU-2007, 2011]

Sol^n

(i) $\because$ $(x-x)$ is divisible by 3 i.e $\frac{0}{3}$

$\therefore (x, x) \in R \quad \forall\ x \in X$

i.e. xRx

So R is reflexive

(ii) Let ~~(x, y) ∈~~ $x, y \in X$ and $(x, y) \in R$

$\Rightarrow (x-y)$ is divisible by 3

$\because \frac{6}{3} = 2$ | $\frac{x-y}{3} = n_1$; $x-y = 3n_1$

$6 = 2 \times 3$

$\Rightarrow (x-y) = 3n_1$ where $n_1 \in I$

$\Rightarrow y - x = -3n_1$ (mul by -1)

$\Rightarrow \frac{y-x}{3} = -n_1$

Here $\Rightarrow (y-x)$ divisible by 3

$\therefore \Rightarrow (y, x) \in R$

$\therefore (x, y) \in R \Rightarrow (y, x) \in R$

or $xRy \Rightarrow yRx \Rightarrow R$ is symmetric

(iii) Transitive – Let $x, y, z \in A$

If $(x,y) \in R$ and $(y,z) \in R$

$\Rightarrow$ $(x-y)$ is divisible by 3 and $(y-z)$ is divisible by 3

$\Rightarrow$ $x-y = 3n_1$ and $y-z = 3n_2$
where $n_1, n_2 \in I$ (Integer)

$\therefore$ $(x-y)+(y-z) = 3(n_1+n_2)$

$x - z = 3n_3$ where $n_3 = n_1+n_2 \in I$

$\frac{x-z}{3} = n_3$

$\therefore$ $(x-z)$ is divisible by 3
so $(x,z) \in R$

Hence $(x,y) \in R$ and $(y,z) \in R$
so $(x,z) \in R$ this transitive property

Hence R is an equivalance Relation on A

Ex – Let A be the set of all integers and Relation R is defined as
$R = \{(x,y) : x \cong y \pmod{m}, m$ divide $(x-y)\}$ (congruent modulo)
where m is a +ve integer
Prove that R is a equivalence relation?
Also show that if $x_1 \cong y_1$ & $x_2 \cong y_2$ then $x_1+x_2 \cong y_1+y_2$ [AKTU 2008/2017]

$\underline{\text{Sol}^n}$ – $R = \{(x,y) : x \equiv y \pmod m\}$, $(x-y)$ is divisible by m }

(i) **Reflexive** –

Since $x-x$ is divisible by m

$\Rightarrow x \equiv x \pmod m$ // defined by the Question

if this then

$\Rightarrow xRx \quad \forall x \in A$

$\Rightarrow$ Reflexive property

(ii) **Symmetric**

if $(x,y) \in A$ and

Let $(x,y) \in R \Rightarrow x \equiv y \pmod m$ // by the definition

~~and~~

$\Rightarrow x-y$ is divisible by m

if $\frac{6}{2} = 3$ then $\frac{-6}{2}$ we can say that

Hence $\Rightarrow -(x-y)$ " " "

$\Rightarrow (y-x)$ " " "

if this

then $\Rightarrow y \equiv x \pmod m$ it means

$\Rightarrow (y,x) \in R$

$\therefore (x,y) \in R \Rightarrow (y,x) \in R$

i.e. $\boxed{xRy \Rightarrow yRx}$ this is symmetric property

(iii) <u>Transitive</u> — Let $x, y, z \in A$, and $(x,y) \in R$ & $(y,z) \in R$ then $(x,z) \in$

By the defination.

$\Rightarrow$ if $(x,y) \in R$ & $(y,z) \in R$

$\Rightarrow x \equiv y \pmod{m}$ & $y \equiv z \pmod{m}$

$\Rightarrow (x-y)$ is divisible by m & $(y-z)$ is divisible by m.

$\Rightarrow (x-\cancel{y}) + (\cancel{y}-z)$ is divisible by m

$\therefore \frac{6}{2}$ and $\frac{8}{2}$ Then we can do like $\frac{6+8}{2}$

$\Rightarrow (x-z)$ is divisible by m

$\Rightarrow (x,z) \in R$

so if $(x,y) \in R$ and $(y,z) \in R \Rightarrow (x,z) \in R$

~~#~~ it means xRy $yRz \Rightarrow xRz$.

This is transitive property

Hece R is equivalence relation.

Ex Let $A = R \times R$ (R is the set of real no.)
and define the following relation on A
$(a, b)\ R\ (c, d) \Rightarrow$ if $\boxed{a^2+b^2 = c^2+d^2}$ condition
verify that (A, R) is an equivalence Relation
[AKTU 2002, 2008, 2016]

Sol^n – (i) Reflexive.
$\forall (a, a) \in A \Rightarrow R \times R$
we have $a^2+a^2 = a^2+a^2$ by the condition
$\Downarrow$
$(a, a)\ R\ (a, a)\ \forall (a, a) \in A$
so R is reflexive.

(ii) Symmetric –
Let $(a, b)\ R\ (c, d)$
$\Rightarrow a^2+b^2 = c^2+d^2$
$\Downarrow$
$c^2+d^2 = a^2+b^2$
$\Rightarrow (c, d)\ R\ (a, b)$.

symmetric property

(iii) Transitive
Let $(a, b)\ R\ (c, d)$ & $(c, d)\ R\ (e, f)$
$\Rightarrow a^2+b^2 = c^2+d^2$ & $c^2+d^2 = e^2+f^2$
$\Rightarrow a^2+b^2 = e^2+f^2$
if this $(a, b)\ R\ (e, f)$

makes Transitive

So R is equivalence relation.

Composite Relation —

Let A, B and C be three non-empty sets; suppose

R be a relation from $\boxed{A \text{ to } B}$

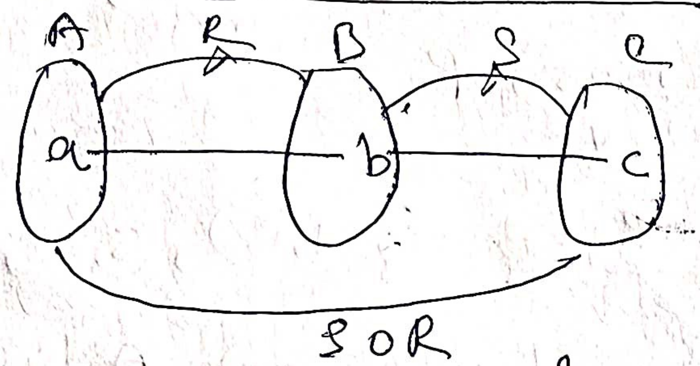
S " " " $\boxed{B \text{ to } C}$ then

the composite relation of the $\boxed{R \text{ and } S}$ is a relation from $\boxed{A \text{ to } C}$ and denoted by $\boxed{S \circ R}$

and defined as

$$\boxed{S \circ R = \{(a, c) : \exists \text{ an element } b \in B \text{ such that } (a, b) \in R \text{ and } (b, c) \in S\}}$$

$\therefore \boxed{(a,b) \in R, (b,c) \in S \Rightarrow (a,c) \in S \circ R}$



Note — $R \circ R = R^2$, $R^2 \circ R = R^3$

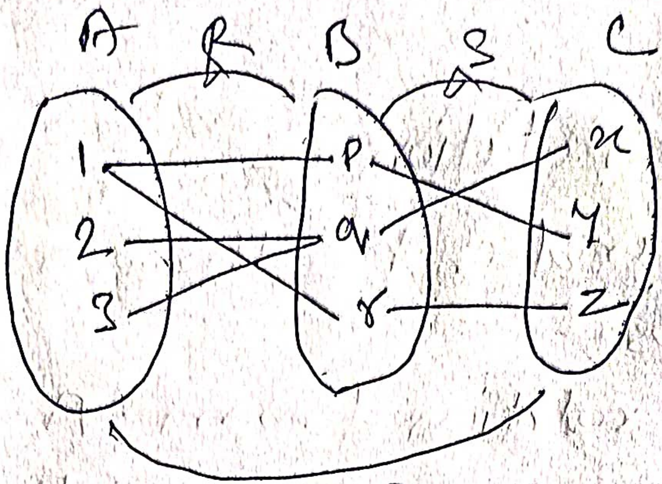
Ex — $A = \{1, 2, 3\}$ $B = \{p, q, r\}$ $C = \{x, y, z\}$

Let $R = \{(1, p), (1, r), (2, q), (3, q)\}$

$S = \{(p, y), (q, x), (r, z)\}$

Calculate $S \circ R$?

Sol^n



from Path

$S \circ R = ?$

$1\ p\ y \Rightarrow (1, y)$
$1\ p\ z \Rightarrow (1, z)$
$2\ q\ x \Rightarrow (2, x)$
$3\ q\ x \Rightarrow (3, x)$

$\Rightarrow S \circ R$

So $S \circ R = \{(1,y), (1,z), (2,x), (3,x)\}$

Ex-2

$R = \{(1,2), (3,4), (2,2)\}$

$S = \{(4,2), (2,5), (3,1), (1,3)\}$

Find (i) $S \circ R$ (ii) $R \circ S$, $R \circ R$ (iii), $R \circ (S \circ R)$ (iv) and $(R \circ S) \circ R$ (v)

Sol^n (i) **$S \circ R$** $= \{(1,5), (3,2), (2,5)\}$

A to B R	B to C S		A to C $S \circ R$
(1, 2)	(2, 5)	$\Rightarrow$	(1, 5)
(3, 4)	(4, 2)	$\Rightarrow$	(3, 2)
(2, 2)	(2, 5)	$\Rightarrow$	(2, 5)

$R \circ S$

$S = \{(4,2), (2,5), (3,1), (1,3)\}$

$R = \{(1,2), (3,4), (2,2)\}$

$R \circ S = \{(4,2), (3,2), (1,4)\}$ Ans

$$\left.\begin{array}{lll} S & R & R \circ S \\ (4,2) & (2,2) \Rightarrow & (4,2) \\ (2,5) & \text{———} & \\ (3,1) & (1,2) \Rightarrow & (3,2) \\ (1,3) & (3,4) \Rightarrow & (1,4) \end{array}\right\}$$

(iii) $R \circ R = R^2$

$R = \{(1,2), (3,4), (2,2)\}$

$R = \{(1,2), (3,4), (2,2)\}$

$\boxed{R \circ R = \{(1,2), (2,2)\}}$ Ans

$$\left.\begin{array}{lll} (1,2) & (2,2) \Rightarrow & (1,2) \\ (3,4) & \text{———} & \\ (2,2) & (2,2) \Rightarrow & (2,2) \end{array}\right\}$$

(iv) $R \circ (S \circ R) \Rightarrow \{(3,2)\}$

$S \circ R = \{\qquad\}$

$R = \{\qquad\}$

(v) $(S \circ R) \circ R = \{(3,2), \qquad\}$

$R = \{\qquad\}$

$S \circ R = \{\qquad\}$

Reversal Law in composite relation —

Problem — Let R be a relation from A to B and S be a relation from B to C then Prove that $\boxed{(S o R)^{-1} = R^{-1} o S^{-1}}$

Proof —

Let $(c, a) \in (S o R)^{-1}$

$\Leftrightarrow (a, c) \in (S o R)$ $\forall$ $a \in A$, $c \in C$

$\Leftrightarrow \exists$ an element $b \in B$ s.t $(a, b) \in R$ & $(b, c) \in S$

$\Leftrightarrow (b, a) \in R^{-1}$ & $(c, b) \in S^{-1}$

$\Leftrightarrow (c, b) \in S^{-1}$ & $(b, a) \in R^{-1}$

$\Leftrightarrow (c, a) \in R^{-1} o S^{-1}$

$\therefore (c, a) \in (S o R)^{-1} \Leftrightarrow (c, a) \in R^{-1} o S^{-1}$

Hence $\boxed{(S o R)^{-1} = R^{-1} o S^{-1}}$ Proved

Order of Relation

Partial order Relation / Total order

Partial order Relation ⇓

A relation R on set A is called partial order relation if R is →

(i) Reflexive (ii) Antisymmetric (iii) Transitive

(i) Reflexive ⇓

$(a,a) \in R \ \forall \ a \in A$

i.e. $aRa \ \forall \ a \in A$

(ii) Antisymmetric ⇓

$(a,b) \in R$ & $(b,a) \in R$

$\Rightarrow a=b \ \forall \ a,b \in A$

(iii) Transitive ⇓

$(a,b) \in R$ & $(b,c) \in R \Rightarrow (a,c) \in R \ \forall \ a,b,c \in A$

$\boxed{aRb \ \& \ bRc \Rightarrow aRc}$

POSET (Partial order Set) —

The set A together with a partial order relation R on the set A, partial order set (POSET).

i.e. (A, R) is partial order set

Ex Show that

$R = \{(x,y) : x \geq y\}$ where $x, y \in I^+$

set of +ve int

is a partial order relation?

Soln — Let $x, y, z \in I^+$

and given $R = \{(x,y) ; x \geq y\}$

(i) Reflexive

Let $x \in I^+$

$\because x \geq x \Rightarrow (x, x) \in R \; \forall x \in I^+$

$\Rightarrow xRx \; \forall x \in I^+$

$\Rightarrow$ This is Reflexive property

(ii) Anti-symmetric

Let $x, y \in I^+$

Now if ~~[illegible]~~ $(x, y) \in R$ & $(y, x) \in R$

$\Rightarrow x \geq y$ & $y \geq x$ possible

if $\Rightarrow x = y$ it prove that

$\Rightarrow (x, y) \in R$ & $(y, x) \in R \Rightarrow \boxed{x = y}$

(iii) Let $x, y, z \in I^+$

Let $(x, y) \in R$ & $(y, z) \in R$

$\Rightarrow x \geq y$ & $y \geq z$

$\Rightarrow x \geq z$

$\Rightarrow (x, z) \in R$

$5 \geq 3$

$3 \geq 2$

$5 \geq 2$

Hence $(x, y) \in R$ & $(y, z) \in R \Rightarrow (x, z) \in R$

So R is transitive

Hence R is a partial order relation on I^+

So (I^+, R) is POSET.

Total order relation —

Consider the relation R on set A. If $\forall a, b \in A$, we have

$$\left.\begin{array}{lll} \text{either} & (a,b) \in R \\ \text{or} & (b,a) \in R \\ \text{or} & a = b \end{array}\right\}$$ Then R is Total order relation on set A.

<u>Ex</u> Show that the relation '<' defined on N, the set of +ve integers is neither an equivalence relation nor partial order relation but is a total order relation.

<u>Soln</u> — Let $a, b \in N$ (+ve integer of natural no.)

$R = \{(a,b) \mid a < b\}$

<u>Reflexive</u> since $a \nless a$

$\Rightarrow (a,a) \notin R \Rightarrow a \not R a$

$\Rightarrow$ R is not reflexive

if R is not reflexive then it is not an equivalence relation and not in partial order relation.

But

But as

$\forall$ $a, b \in N$ we have

either $a < b \Rightarrow (a, b) \in R$

or $b < a \Rightarrow (b, a) \in R$

or $a = b \Rightarrow a = b$

So the relation is in total order relation.